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# The evolution of stellar exponential discs

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## ABSTRACT

Models of disc galaxies which invoke viscosity-driven radial flows have long been known to provide a natural explanation for the origin of stellar exponential discs, under the assumption that the star formation and viscous time-scales are comparable. We present models which invoke simultaneous star formation, viscous redistribution of gas and cosmologically-motivated gaseous infall and explore the predictions such models make for the scalelength evolution and radial star formation history of galactic stellar discs. While the inclusion of viscous flows is essential for ensuring that the stellar disc is always exponential over a significant range in radius, we find that such flows play only a minor role in determining the evolution of the disc scalelength. In models in which the main infall phase precedes the onset of star formation and viscous evolution, we find the exponential scalelength to be rather invariant with time, with the bulk of the disc stars at all radii out to  $\sim 5$  scalelengths being relatively old (i.e. ages  $\gtrsim 6$ – $8$  Gyr for an assumed disc age of 11 Gyr). On the other hand, models in which star formation/viscous evolution and infall occur concurrently result in a smoothly increasing scalelength with time, reflecting the mean angular momentum of material which has fallen in at any given epoch. The disc stellar populations in these models are predominantly young (i.e. ages  $\lesssim 5$  Gyr) beyond a few scalelengths. In both cases, viscous flows are entirely responsible for transporting material to very large radii. Our results are robust for a range of currently popular star formation laws and infall prescriptions. We discuss existing observational constraints on these models from studies of both local and moderate redshift disc galaxies. In particular, a good agreement is found between the solar neighbourhood star formation history predicted by our infall model and the recent observational determination of this quantity by Rocha-Pinto et al.

**Key words:** galaxies: evolution – galaxies: formation – galaxies: fundamental parameters – galaxies: spiral – galaxies: structure.

## 1 INTRODUCTION

One of the most notable properties of disc galaxies is that the surface brightness profiles of their stellar discs are remarkably close to exponential distributions over a large dynamic range in radii (e.g. de Vaucouleurs 1959; Freeman 1970; de Jong 1996a). This has traditionally been held to imply an exponential surface density profile for gas infalling to the galactic plane, which in turn places particular constraints on the detailed angular momentum distribution of baryonic material in the protogalaxy. Gunn (1982), for example, showed that, under the assumption of angular momentum conservation for each gas element under infall, a uniformly rotating uniform sphere collapses to an approximately

exponential profile, however the resulting function is only exponential over  $\sim 3$  scalelengths compared to the more impressive 4–6 scalelengths exhibited by real galaxies (e.g. de Jong 1996a). Calculations with other initial halo density profiles and angular momentum distributions produce similar results (e.g. van der Kruit 1987; Dalcanton, Spergel & Summers 1997; Bullock et al. 2000).

The exponential profile is so ubiquitous, and its appearance when plotted as magnitude versus radius so deceptively simple, that one may be tempted to regard it as a natural or obvious outcome of gravitational collapse. In fact, gravitational collapse calculations in other astronomical contexts never give rise to exponential disc profiles (Cassen & Moosman 1981; Terebey, Shu & Cassen 1984), with the more usual outcome being profiles of approximately power law form. In the galactic context, the difficulty of producing exponential profiles over a large number of scalelengths through angular momentum conserving collapse from reasonable initial conditions has recently been highlighted by

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Efstathiou (2000): the outer regions of galactic discs contain so little mass, that for power law halo density profiles, the outer disc material must originate from a small range of radii in the halo. On the other hand, these regions contain a large dynamic range of angular momentum, so that these conditions can only be met simultaneously if the angular momentum profile at the halo suffers an abrupt, and physically implausible, upturn at some radius (see fig. 6b of Efstathiou 2000).

Exponential profiles have the unique feature that the scalelength on which quantities change is the same at all radii. In the case of a centrifugally-supported disc, the exponential scalelength is determined (in a fixed potential) purely by the average angular momentum per unit mass for the entire disc. Such a system thus has the remarkable property that material over a significant range in radii (and with a correspondingly significant range in specific angular momenta) is somehow ‘informed’ about the average specific angular momentum of the disc as a whole. A key question is whether this property is a result of the formation process of the disc, or somehow acquired through subsequent evolution.

Disc formation models have moved on from the picture of smooth dissipational collapse within a dark halo potential, and recent numerical calculations have focused on the more complex formation histories resulting from hierarchical growth (e.g. Kauffmann, White & Guiderdoni 1993; Cole et al. 1994; Navarro, Frenk & White 1995). A notable outcome of such calculations is the strong redistribution of angular momentum during collapse. Attention has so far been mainly focused on the net transfer of angular momentum from baryonic to dark matter and the consequent implications for the overall sizes of galactic discs (e.g. Navarro & Benz 1991; Navarro & Steinmetz 1997; Weil, Eke & Efstathiou 1998). A related issue is, however, the redistribution of angular momentum within the baryonic material and its effect on the resulting disc profile. The large dynamic range in size scales that have to be captured in such simulations, and the difficulties in controlling the effective viscosity in smoothed-particle hydrodynamical (SPH) calculations that are close to the limit of adequate resolution, mean that it is not yet clear whether discs that are exponential over a number of scalelengths are indeed the expected outcome.

Observational evidence against a picture of exponential infall followed by *in situ* star formation is provided by the radial profiles of gas in disc galaxies. The gas surface density profiles are generally observed to be much flatter than the stellar profiles. In the inner regions of disc galaxies, the gas is the minority component and represents the residue following star formation: it is not difficult, by appropriate adjustment of the star formation law, to generate gas profiles within the optical disc that are compatible with observations using an initially exponential gas profile (e.g. Boissier & Prantzos 1999). In the outer regions, however, the gas is the dominant component and its profile should presumably reflect its infall distribution. It seems highly suspicious, in the context of *in situ* models, that the gas profiles at large radii are so different from the exponential profiles that characterise the stars at smaller radii: if the stellar exponential profile derives from exponential infall of gas, surely one would expect to see that profile continued in the gas not yet incorporated in stars?

All of these difficulties are circumvented by the simple model first proposed by Lin & Pringle (1987a). These authors envisaged that gas in the disc not only turns into stars (on a time-scale  $t_*$ ), but is also subject to some form of effective viscosity that gives rise to angular momentum redistribution and radial flows. The dominant physical process responsible for the viscosity is poorly constrained,

but could plausibly result from cloud–cloud collisions and/or gravitational instabilities in the disc. If viscous redistribution occurs on a time-scale  $t_v$ , then the resultant stellar profile is exponential for a wide range of initially less centrally concentrated gas profiles, provided only that  $t_v \sim t_*$ . This result can be understood, in the context of the above discussion, in as much as viscous processes in the disc (provided  $t_*$  is not much less than  $t_v$ ) allow material at different specific angular momenta to communicate with each other, and thus to ‘inform’ themselves of the average specific angular momentum of disc material, which in turn sets the disc scalelength. On the other hand,  $t_*$  cannot be much greater than  $t_v$ , because, in that case, viscous evolution would run away too far towards its ultimate endpoint [i.e. ‘all the mass at the origin, all the angular momentum at infinity’, Lynden-Bell & Pringle (1974)] to be compatible with the observed profiles of disc galaxies. Fortunately it turns out that the generation of exponential profiles over a number of scalelengths does not require particularly fine tuning between  $t_*$  and  $t_v$ , and that satisfactory fits are obtained even if the ratio  $t_v/t_*$  deviates from unity by half an order of magnitude, or is a mild function of radius (Clarke 1989).

The viscous model thus relieves cosmological models of the burden of generating the ‘right’ angular momentum profile (though, by the same token, it also erases information about the profile on infall, and thus weakens the role of gaseous and stellar disc profiles as cosmological probes). It also predicts that the radial gas surface density distribution beyond the optical radius should be rather flat, in accordance with observations. The viscous model was able to satisfy the various observational constraints available a decade ago (mainly surface density profiles and chemical constraints imposed by the Milky Way), although it was not manifestly superior to conventional *in situ* models in this respect (Clarke 1989; Yoshii & Sommer-Larsen 1989). Then, as now, the main drawback to viscous models is the lack of a detailed theory linking  $t_*$  to  $t_v$ , although it is not difficult to invoke physical processes (e.g. self-gravitational instability of the disc) that one might expect to promote star formation and angular momentum redistribution in the disc simultaneously (Lin & Pringle 1987b; Olivier, Primack & Blumenthal 1991).

A wealth of new observational data pertaining to the evolution of disc galaxies has emerged in recent years from both high redshift galaxy surveys – which permit a direct study of how galactic properties evolve with epoch – as well as detailed studies of the fossil stellar populations in our own Milky Way and other local systems. Given that the general arguments in favour of viscous models appear at least as strong now as they did a decade ago, it would seem timely to revisit this class of model and investigate its predictions in the light of new and forthcoming observations. In this paper, we explore the structural evolution and radial growth of stellar discs subject to simultaneous star formation and viscous evolution by considering models in which the initial gas disc is pre-assembled as well as models in which the gas disc is assembled over an extended period due to the continued infall of material from the protogalactic halo. In both cases, the gaseous and stellar discs are naturally built up from the inside-out, though the mechanism is different in each case: in pre-assembled models, gas is transferred to larger radii only as a result of viscous torques in the disc, whereas in the case of continued infall, the dominant cause of the inside-out growth is the increasing specific angular momentum of infalling material at later times (whether or not viscous flows are also important).

We do not address in this paper how well viscous models can be made to reproduce in detail the present-day properties of the Milky

Way and other nearby spirals; for a recent discussion of this aspect see Thon & Meusinger (1998). We also bypass the issue of how one sets up the conditions in the collapsing gas that will reproduce the correct *absolute* values of the disc scalelengths. The resultant scalelength in viscous evolution models is set purely by the average specific angular momentum of the gas infallen at a given time. Hence the problem of forming disc systems with large scalelengths (as discussed by Mo, Mao & White 1998) is one of the cosmological initial conditions and not one that viscous evolution can alleviate or affect in any way. In the present paper, we simply assume that the forming disc has access to material of the requisite high angular momentum. We describe in Section 2 the model and free parameters and present our results in Section 3. Section 4 discusses model constraints from existing and future observational data on both local and distant spirals.

## 2 MODEL PARAMETERS

We envisage a general scenario for disc galaxy formation in which infalling material gradually builds up the disc over a time interval  $t = 0$  to  $t = t_{\text{in fall}}$  and (simultaneous) star formation and viscous redistribution occur over the interval  $t = t_{\text{sf}}$  ( $\leq t_{\text{in fall}}$ ) to  $t = t_{\text{now}}$  ( $\geq t_{\text{in fall}}$ ). The case  $t_{\text{sf}} = t_{\text{in fall}}$  corresponds to the case of star formation in a pre-assembled gas disc whereas  $t_{\text{now}} \sim t_{\text{in fall}} \gg t_{\text{sf}}$  corresponds to the extreme infall case where gas is accreted up until the present epoch.

### 2.1 Infall calculation

We calculate the surface density profile with which material arrives in the disc plane by assuming an idealised (spherically symmetric) density distribution and angular velocity distribution for baryonic material in the protogalaxy. We then map this material down into the disc plane by invoking detailed angular momentum conservation, neglecting both the self-gravity of the baryonic material and the response of the halo to the infalling baryons. We justify this rigid halo approximation on the grounds that the initial distribution of baryons in the halo is poorly known (and likely to be considerably more complex than the smooth spherically symmetric distribution invoked here) and also because, in any case, the behaviour of the viscous evolution is only weakly dependent on the shape of the initial distribution (as opposed to its total mass and angular momentum). We calculate the gas infall rate (for  $t \leq t_{\text{in fall}}$ ) by requiring that all material infalling from within a spherical radius  $r_{\text{inf}}(t)$  has reached the disc plane by a time  $t$ . This infall radius is fixed by the requirement that the free-fall time within  $r_{\text{inf}}$  is much less than the instantaneous cosmic shear time ( $H^{-1}$ ) – or, equivalently, that the mean enclosed density is much larger ( $\approx 200$ ) than the mean density of the Universe at that epoch. As successive shells of matter are ‘released’ and allowed to fall in, we map them onto the disc plane [without applying the (small) correction for the finite infall time of each shell]. We do not explicitly consider the gas cooling time in our model; numerical simulations suggest that the cooling radius tracks the virial radius for Milky Way-type galaxies and hence that all gas within  $r_{\text{inf}}$  is indeed able to infall on a free-fall time (Somerville & Primack 1999). Our default halo model assumes that the baryons initially follow an  $r^{-2}$  density distribution (i.e. isothermal sphere), so that for an Einstein–de Sitter cosmology, the total infall rate onto the galaxy is *constant* in time for  $t < t_{\text{in fall}}$ . At any given radius however, the infall history is characterized by an initial lag of variable length (corresponding to the time required for a shell of high enough angular momentum to

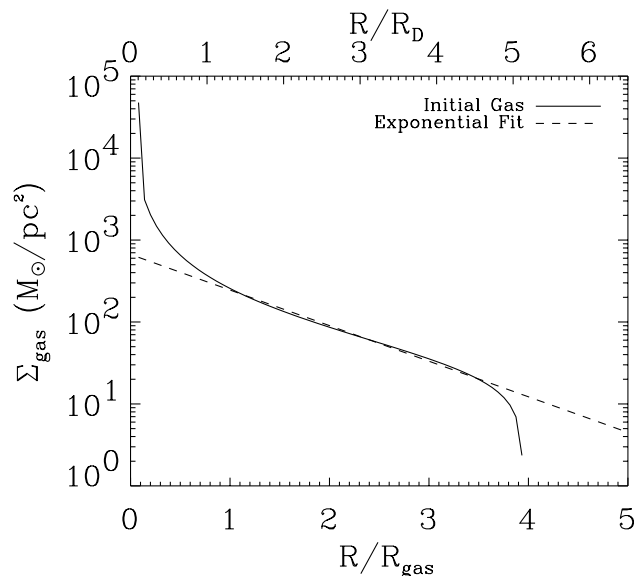
fall in), a peak and then a roughly exponential decline. This prescription thus produces a crude on/off switch on the infall, so that, if  $t_{\text{in fall}}$  is comparable to  $t_{\text{now}}$ , it implies considerably more infall at recent times and at large radii than is usually assumed in galactic infall models, where the infall is *arbitrarily* assumed to decline exponentially in both time and radius (e.g. Chiappini, Matteucci & Gratton 1997; Boissier & Prantzos 1999) in order to mimic inside-out growth. We motivate the prescription employed here by the wish to explore the response of viscous discs to extreme infall conditions.

Finally, in order to prescribe the surface density distribution generated by each shell as it falls in, we must assign an initial angular momentum distribution for the baryons in the halo. The angular momentum of the dark matter and baryons is presumably acquired through tidal torques from neighbouring structures (Peebles 1969) and thus the average angular momentum of infalling material (whether the infall be smooth or in the form of discrete clumps) is expected to increase with time. We assume the halo baryons initially rotate rigidly on spheres with angular velocity  $\propto r^{-1}$ , which implies that the mean value of the spin parameter  $\lambda$ , where

$$\lambda = J|E|^{1/2}M^{-5/2}G^{-1},$$

for each shell is constant. For a power law angular velocity law, the surface density distribution generated by each shell is self-similar. We scale the overall dimensions and normalization of the system to match the observed profile of the Milky Way. As stated before, our calculation thus sidesteps the difficulty of how, given the usual assumptions about the value of  $\lambda$  and the size scale for collapse as a function of redshift, it is possible to produce large, Milky Way-scale, discs at redshifts greater than  $\sim 1$  (e.g. Mo et al. 1998).

Fig. 1 shows the final gas surface density profile at the



**Figure 1.** The final gas surface density profile at the completion of the infall phase (solid line) – before star formation switches on – for a singular isothermal halo. The bottom radial coordinate is normalized to the scalelength,  $R_{\text{gas}}$  derived from an exponential fit to the intermediate region of the profile (dashed line). Note that an exponential provides a good fit to the profile only over  $\sim 2.5$  scalelengths. The top radial coordinate is normalized to the scalelength of the stellar disc which ultimately forms from this gas distribution, for the case of a star formation law  $\propto \Sigma_{\text{gas}} R^{-1}$ . For the halo profile and rotation law adopted here,  $r_{\text{init}} \sim 5 R_D$ .

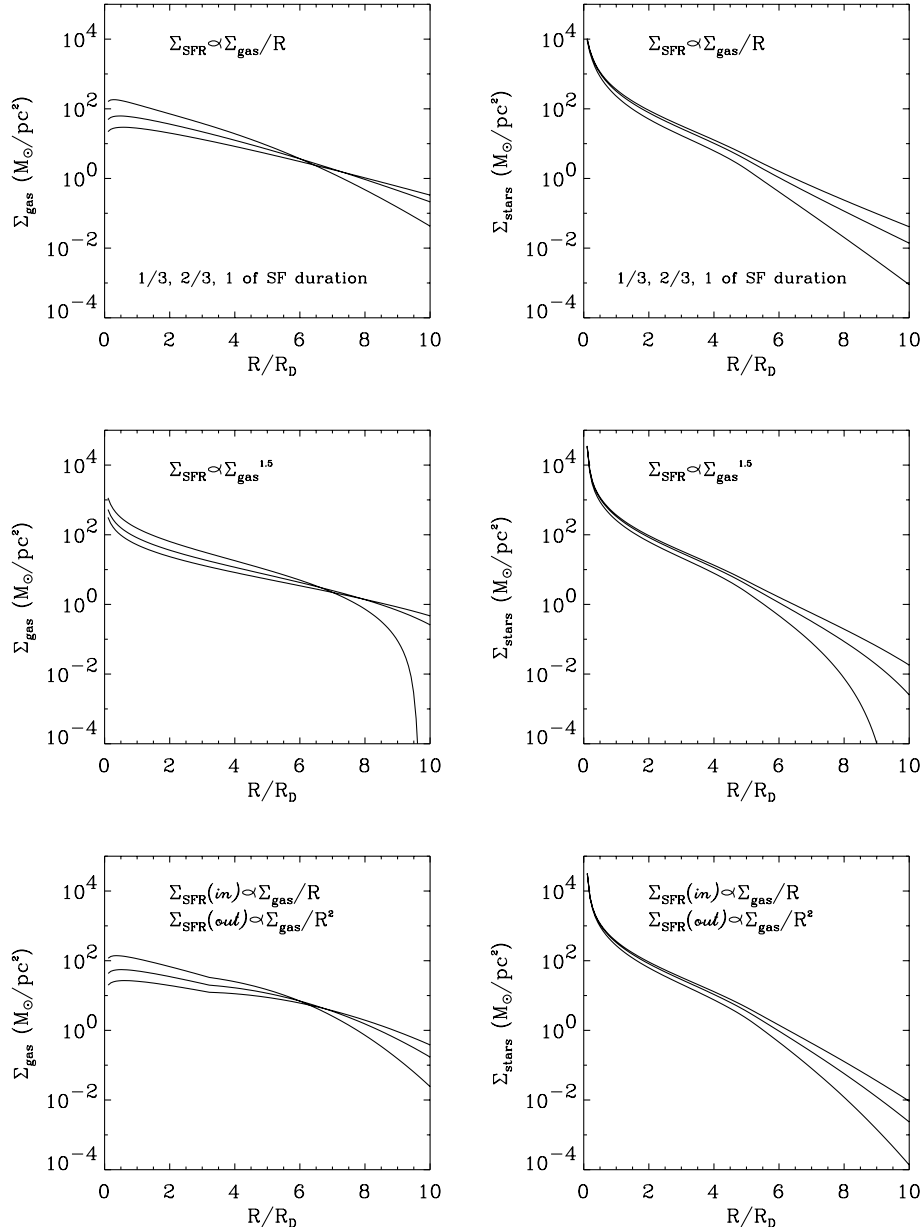
completion of the infall phase (hence the initial starting point in the case of a pre-assembled gas disc). Beyond the central region, the gas surface density slowly declines and then turns over abruptly at the maximum infall radius,  $r_{\text{init}}$ . The value of  $r_{\text{init}}/R_D$  depends primarily on the initial halo profile and rotation law, and to a lesser extent on the star formation and viscosity prescriptions. If the disc surface density decreases outwards throughout the disc,  $r_{\text{init}} > 3 R_D$ ; the absolute minimum value of  $r_{\text{init}}$ , which corresponds to an annulus of matter at this location, is  $2 R_D$ . This latter configuration does not viscously evolve to a smooth exponential profile however. For the flat rotation curve and halo profile assumed here (and neglecting disc self-gravity),  $r_{\text{init}} \sim 5 R_D$ . An exponential fit (dashed line) provides a good description of the radial behaviour of the initial gas profile over  $\sim 2.5$  scalelengths.

## 2.2 Disc evolution

The evolution of gas surface density,  $\Sigma_{\text{gas}}(r, t)$ , in a disc that is subject to simultaneous viscous redistribution and star formation is described by:

$$\frac{\partial \Sigma_{\text{gas}}}{\partial t} = -\frac{1}{r} \frac{\partial}{\partial r} \left\{ \frac{\partial / \partial r [\nu \Sigma_{\text{gas}} r^3 (d\Omega/dr)]}{d/dr(r^2 \Omega)} \right\} - \frac{\Sigma_{\text{gas}}}{t_*} \quad (1)$$

where  $r$ ,  $\Omega$ ,  $\nu$  and  $t_*$  represent the galactocentric radius, angular velocity, kinematic viscosity and local star formation time-scale, respectively. The evolution of the stellar surface density  $\Sigma_*(r, t)$  is



**Figure 2.** Snapshots of the gaseous (left) and stellar (right) surface density distributions at various fractional times (1/3, 2/3 and 1) of the star formation duration (which terminates at  $t_{\text{now}}$ ) for the case of a pre-assembled gas disc. The radial coordinate is normalized to the stellar exponential scalelength measured at the end of the model. The final solutions are those with the highest stellar profile at all radii and the lowest inner gas profile. The initial gas disc in these models extends to approximately 5 (final) stellar exponential scalelengths. Different star formation laws are shown in top, middle and bottom panels.



then given by:

$$\frac{\partial \Sigma_*}{\partial t} = \frac{\Sigma_{\text{gas}}}{t_*} \quad (2)$$

The gas surface density in these models is assumed to be the sum of the neutral and molecular components. We calculate the evolution of the gaseous and stellar components of the disc, for times  $t > t_{\text{sf}}$ , assuming that the gas evolves in the *fixed* potential of a halo with flat rotation law (i.e.  $\Omega \propto r^{-1}$ ). This simplification will not be correct for the very inner regions of the disc, and so we do not attempt any detailed comparison of our results with structures in the central parts of disc galaxies.

In order to obtain a resulting exponential stellar profile, we follow Lin & Pringle (1987a) and fix the relationship between the star formation and viscosity prescriptions such that the characteristic time-scale for material to move a fractional radius of order unity is comparable with the time-scale on which it is converted into stars (i.e.  $t_* \sim t_\nu$ ). With this constraint, and given an assumed rotation law, the only free parameter is the star formation prescription. Our default star formation law relates the star formation rate (SFR) per unit area with the gas column density,  $\Sigma_{\text{gas}}$ , as  $\Sigma_{\text{SFR}} \propto \Sigma_{\text{gas}}^{-1}$ , which corresponds to an  $n = 1$  Schmidt law, modulated by the local angular frequency (e.g. Wyse & Silk 1989). A prescription of this sort (with power law  $n = 1.5$ ) has recently been shown to be in good agreement with the observed radial SFR in the Milky Way (Boissier & Prantzos 1999). We also consider a model with a dependence of  $\Sigma_{\text{SFR}} \propto \Sigma_{\text{gas}}^{1.5}$  as well as a model in which the star formation law steepens considerably in the outer disc,  $\Sigma_{\text{SFR}}(R > 3R_D) \propto \Sigma_{\text{gas}}/R^2$ . While the former has been shown to provide a reasonable description of the SFRs observed in the inner regions of galactic discs, the two-component law may provide a better description of the steep declines in SFR observed at large radii, perhaps resulting from the flaring of the gas disc in these parts (e.g. Ferguson et al. 1998). We stress that in this paper we are seeking generic properties of viscous models, and their inter-relation with continued infall, that do not depend strongly on the star formation prescription employed. While varying the star formation prescription also requires changing the form of the viscosity law so that the approximate equality in the respective time-scales is preserved, the dominant viscosity-producing mechanism in discs is so poorly constrained that this does not create any obvious physical problem.<sup>1</sup>

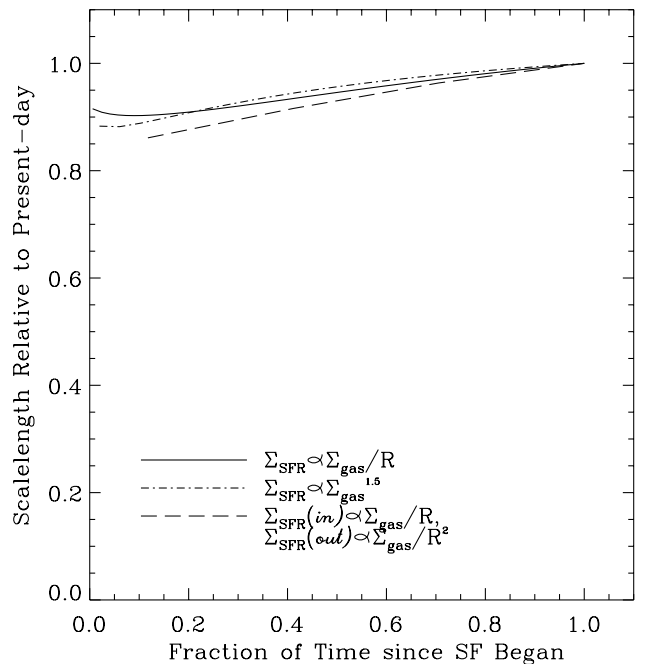
We compute the evolution of a viscous star-forming disc subject to infall using a standard first-order explicit scheme with 200 gridpoints, equispaced in  $R$  with  $R_{\text{out}}/R_{\text{in}} = 150$ . Zero torque boundary conditions are applied at the inner and outer edge of the grid. In practice, we prescribe  $t_{\text{sf}}$  and  $t_{\text{in fall}}$  and then pursue the calculation until a time  $t_{\text{now}}$  at which the system is deemed to resemble the Milky Way – specifically, we require that the gas fraction at a radius of 2.7 exponential scalelengths<sup>2</sup> is  $\approx 0.4$ .

### 3 RESULTS

#### 3.1 The case of a pre-assembled gas disc

We first consider the results of models in which star formation (hence viscous redistribution) switches on after the initial gas disc is already in place – i.e. pre-assembled. This corresponds to the

scenario where all or most of the infall phase is complete before the onset of star formation ( $t_{\text{sf}} \geq t_{\text{in fall}}$ ). Fig. 2 shows snapshots of the gaseous and stellar surface density distributions at various fractional times since the onset of star formation for three different star formation prescriptions ( $\Sigma_{\text{SFR}} \propto \Sigma_{\text{gas}}/R$ ,  $\Sigma_{\text{SFR}} \propto \Sigma_{\text{gas}}^{1.5}$  and a ‘two-component’ law described by  $\Sigma_{\text{SFR}} \propto \Sigma_{\text{gas}}/R^n$ , where  $n = 1$  out to 3 scalelengths and  $n = 2$  beyond that). The profiles are normalized by matching the final (i.e. present day) gas and stellar surface density in the solar neighbourhood to their measured values (we assume  $\Sigma_{\text{gas}} = 14.5 \text{ M}_\odot \text{ pc}^{-2}$  (Olling & Merrifield 1998),  $\Sigma_{\text{stars}} = 40 \text{ M}_\odot \text{ pc}^{-2}$  (Gould, Bachall & Flynn 1996). Fig. 2 illustrates several notable features. First, different star formation prescriptions lead to very similar exponential behaviours (over at least 5 scalelengths) for the stellar surface density profiles; models really only deviate from each other at early times and at large (unobservably faint) radii. Interestingly, the two-component model produces rather smooth profiles despite the abrupt change in the form of the star formation and viscosity laws at roughly 3 scalelengths. These particular profiles deviate somewhat from exponential behaviour beyond 5–6 scalelengths, but the effect is subtle. Gas profiles are more sensitive to the different star formation laws, but generally exhibit exponential declines in their outer regions with e-folding scales significantly larger than that of the stars. At late times, the scalelength of the gas at large radii, where the gas is still the majority component, exceeds the stellar scalelength by a factor of approximately 2. Secondly, we find the stellar component exhibits self-similar growth out to  $\sim 5$  scalelengths over the entire star-forming period, implying an invariance in the stellar scalelength with time, as first noted by Olivier et al. (1991). This is further quantified in Fig. 3, which shows that the scalelength lies well within  $\sim 20$  per cent of its present-day value throughout the history of the disc. Thus, while



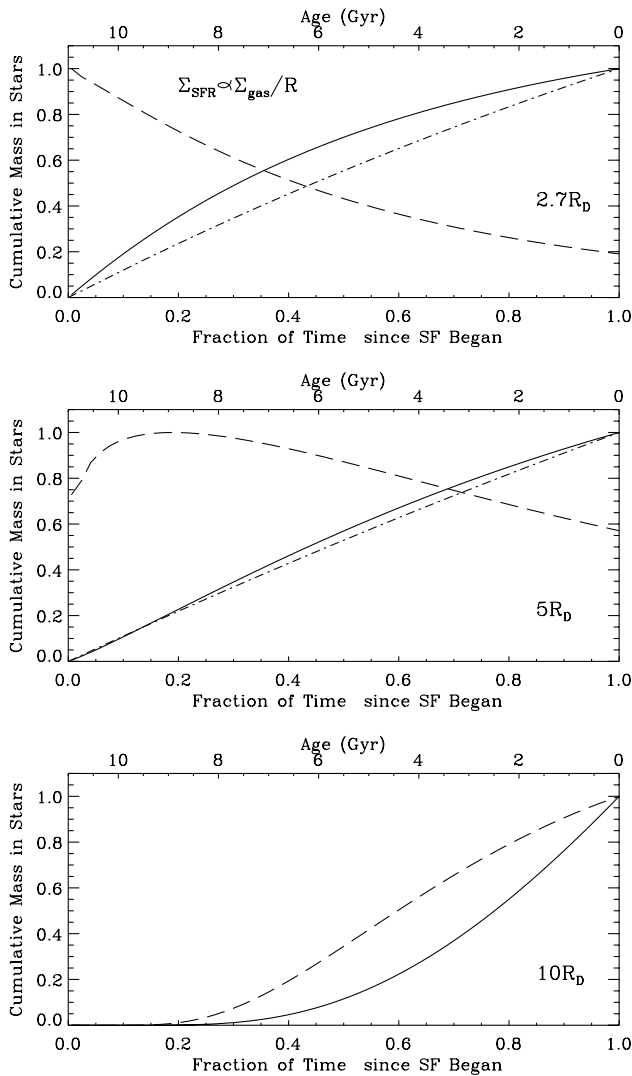
**Figure 3.** Evolution of the stellar exponential scalelength relative to the present-day value as a function of time since the onset of star formation for the case of a pre-assembled gas disc. Results for different star formation laws are shown. At each epoch, the stellar scalelength is measured by fitting the surface density profile over the region corresponding to  $\sim 0.3$ – $0.9$  of the radius of the initial gas disc.

<sup>1</sup>For a given star formation law of the form  $\Sigma_{\text{SFR}} \propto \Sigma_{\text{gas}}^a R^{-b}$ , the corresponding viscosity prescription is  $\nu \propto \Sigma_{\text{gas}}^{a-1} R^{2-b}$ .

<sup>2</sup>We adopt  $R_\odot = 8.0 \text{ kpc}$  (Reid 1993) and  $R_D = 3.0 \text{ kpc}$  (Sackett 1997).

viscous discs are characterized by inside-out growth, which has led to the speculation that the resulting scalelength should increase with time (e.g. Zhang & Wyse 2000), we show here that this growth is simply manifest as an expansion in the radial extent of the exponential stellar disc due to the viscous transport of gas from smaller radii. This effect is unlikely to be verifiable by studies of high redshift galaxies, however, since the strongest evolution occurs in regions of the disc so faint that they are inaccessible to direct observation.

Fig. 4 shows the SFR (dashed lines) normalized to the peak rate as a function of time at 2.7 (‘solar neighbourhood’), 5 and 10 exponential scalelengths for the particular star formation law  $\Sigma_{\text{SFR}} \propto \Sigma_{\text{gas}}/R$ ; other star formation prescriptions exhibit similar behaviours. For a Freeman exponential disc ( $\Sigma_B = 21.65 \text{ mag arcsec}^{-1}$ ), 5 scalelengths corresponds to an expected



**Figure 4.** The star formation history (dashed line) of the disc at radii corresponding to 2.7 (top), 5 (middle) and 10 (bottom) exponential scalelengths in the case of a pre-assembled gas disc. The SFR is normalized to the peak rate at each radius. These models are for a star formation law  $\Sigma_{\text{SFR}} = \Sigma_{\text{gas}}/R$ . The top axis shows the age of the disc, assuming the present-day age of 11 Gyr (Binney et al. 2000). Also shown is the fraction of the final stellar mass in place (solid line) as a function of time and the corresponding analytic prediction of an *in situ* model (dashed-dotted line) obeying the same star formation law. We do not show the *in situ* predictions for the disc at 10 scalelengths, since this lies well beyond the extent of the initial gas disc where gas would never reach in the absence of viscous flows.

surface brightness of  $\sim 1$  per cent of sky, or about the current limit for quantitative studies of surface brightness in external galaxies. The top axis shows age of the disc in Gyr, where we have assumed the present-day stellar disc age of the Milky Way to be 11 Gyr as determined from the ages of the oldest stars in the *Hipparcos* sample (Binney, Dehnen & Bertelli 2000). The disc at 2.7 and 5 scalelengths exhibits slowly declining SFRs in the pre-assembled model, decreasing by factors of roughly 5 and 2 over the lifetime of the disc, respectively. On the other hand, the disc at 10 exponential scalelengths is characterized by a slowly increasing SFR (bottom panel of Fig. 4) reflecting the finite time required to transport gas outwards to these parts by viscous torques; the arrival of material in the extreme outermost disc is therefore delayed long after the onset of star formation at smaller radii. This delay is not a major effect at 5 scalelengths however, even for the most compact plausible initial gas distributions.

Also shown in Fig. 4 is the fraction of the final stellar mass in place as a function of time (solid lines). These curves are contrasted with the analytic predictions of a simple *in situ* model (dashed-dotted line) obeying the same star formation law (note that in the *in situ* model the total – gas + stars – radial profile is exponential by construction at all times) calculated as:

$$\Sigma_{\text{SFR}} = \frac{\partial \Sigma_{\text{gas}}}{\partial t} = \frac{k \Sigma_{\text{gas}}}{R} \quad (3)$$

where the adopted star formation efficiency,  $k$ , is chosen to be consistent with the viscous model. Integration yields

$$\frac{\Sigma_{\text{stars}}(t)}{\Sigma_{\text{stars}}(t_{\text{now}})} = \frac{1 - e^{-kt/R}}{1 - e^{-kt_{\text{now}}/R}} \quad (4)$$

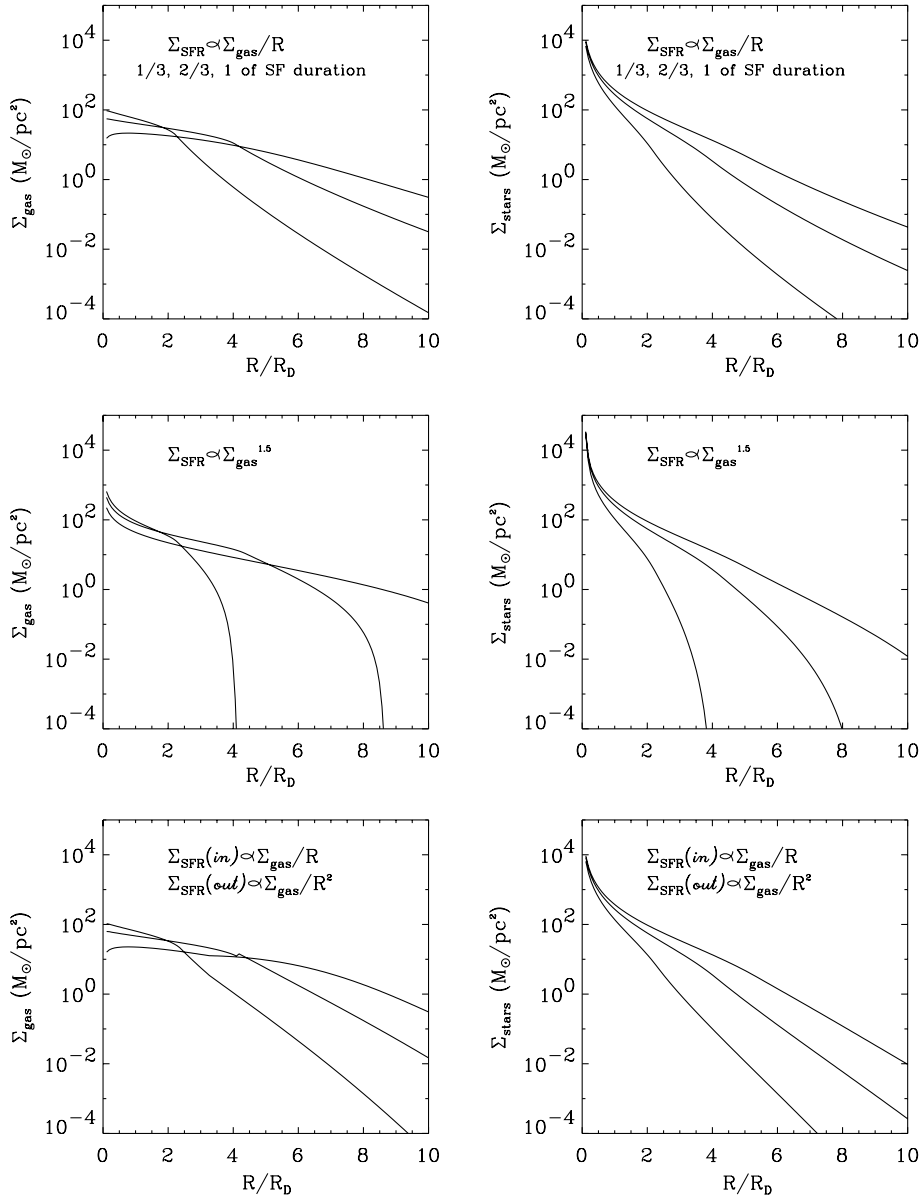
where we have normalized the mass in stars present at a given time to that of the present day. While the mean age of the stellar population present at 5 scalelengths is smaller than at 2.7 scalelengths, the effect is small and the bulk of the stars at this radius have formed early on. Specifically, while 50 per cent of the final stellar surface density is in place after 0.3 of the star formation duration (or  $\sim 3.5$  Gyr for our assumed disc age) at 2.7 scalelengths, this same amount is in place after  $\sim 0.4$  of the star formation period (or  $\sim 5$  Gyr) at 5 scalelengths. Interestingly, there does not appear to be any great difference between the growth rate of stellar mass at 5 scalelengths in the viscous and *in situ* models, implying that viscous flows have little impact on the star formation history at these radii; observations of stellar age distributions alone in these parts could not distinguish between viscous and *in situ* scenarios. Beyond the radius of the initial gas disc, the stellar population becomes increasingly young with radius.

### 3.2 The case of extreme infall

We now contrast the models above with viscous models in which  $t_{\text{in fall}} \sim t_{\text{now}}$ . We refer to this as the ‘extreme infall’ case, since the total mass infall rate onto the galaxy has been constant up to nearly the present day, and star formation in the disc started early on in the infall period ( $t_{\text{star}} \sim \frac{1}{10} t_{\text{in fall}}$ ). Fig. 5 shows the stellar and gaseous surface density profiles for different star formation prescriptions at various times since the onset of star formation. For all models, the final stellar and gaseous profiles are almost identical for the extreme infall viscous model and the pre-assembled viscous model (contrast the highest stellar profiles and lowest central gas profiles in Figs 2 and 5). The temporal evolution of these profiles is very different however. While the disc is able to adjust continuously to the changing profile of the infallen material so as to generate an

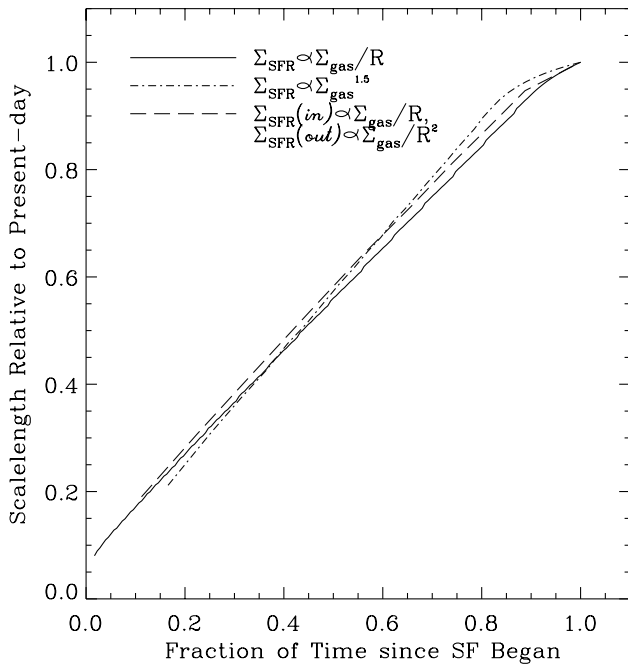
approximately exponential stellar profile at all times, the scalelength of the exponential evolves systematically with time, simply reflecting the mean specific angular momentum of the material which has fallen in by that epoch. The stellar profiles are always exponential over at least 4–5 scalelengths. For the density and angular momentum distributions in the halo assumed here ( $\rho \propto r^{-2}$ ,  $\Omega \propto r^{-1}$ ), the total angular momentum scales with the square of the infallen mass, so that the mean specific angular momentum is simply proportional to the infallen mass. Our default infall prescription, which makes specific assumptions about the halo properties and the cosmological model (see Section 2.1), implies a constant infall rate for  $t \leq t_{\text{in fall}}$  and hence the mean specific angular momentum of the infallen material rises linearly with time over this period. Fig. 6 demonstrates that the stellar scalelength does indeed display this behaviour.

A fundamental assumption in our infall model is that the mean specific angular momentum of the accreted material systematically increases with time. We feel this is a valid assumption, regardless of the exact nature of the infall, and should be preserved. For example, if the dominant mode of infall is high velocity cloud accretion (Blitz et al. 1999), one would still expect those clouds falling in at late times to have originated further from the Milky Way and to have suffered *on average* more tidal torquing than clouds which fell in at early times. While individual clouds will carry a range of angular momenta (and indeed in our models a given shell maps onto a range of radii), the mean trend should be for the average angular momentum to increase with time. It is worthwhile investigating how much of the scalelength evolution is due to other aspects of our adopted infall model however. Fig. 7 shows the growth of the stellar exponential scalelength for an

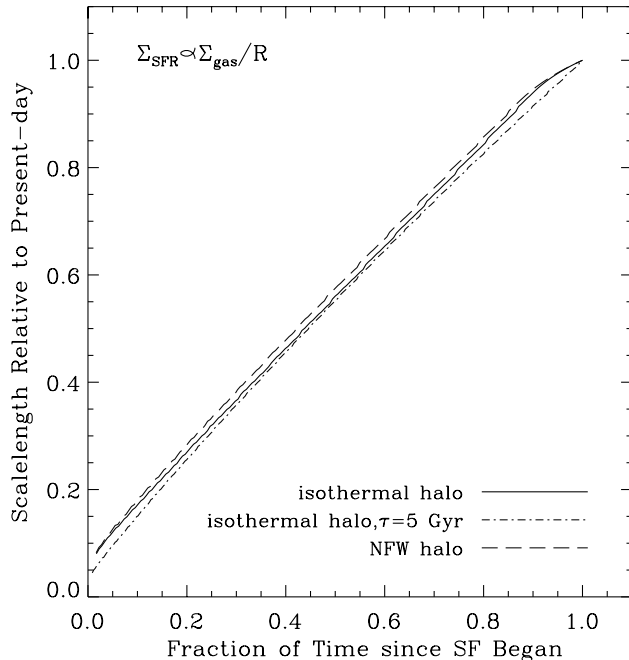


**Figure 5.** Snapshots of the gaseous (left) and stellar (right) surface density distributions at various fractional times (1/3, 2/3 and 1) of the star formation duration (which terminates at  $t_{\text{now}}$ ) for the extreme infall model. In this model star formation starts shortly after the infall phase begins ( $t_{\text{star}} \sim \frac{1}{10} t_{\text{in fall}}$ ). The radial coordinate is normalized to the stellar exponential scalelength measured at the end of the model. The final solutions are those with the highest stellar profile at all radii and the lowest inner gas profile. Different star formation laws are shown in top, middle and bottom panels.





**Figure 6.** Evolution of the stellar exponential scalelength relative to the present-day value as a function of the star formation duration for the constant infall model. Results for different star formation laws are shown. At each epoch, the stellar scalelength is measured by fitting the surface density profile over the region corresponding to 0.3–0.9 of the instantaneous infall radius.



**Figure 7.** Evolution of the stellar exponential scalelength relative to the present-day value as a function of different infall prescriptions (i.e. halo profiles). As in Fig. 6, infall has been occurring in these models since the onset of star formation. At each epoch, the stellar scalelength is measured by fitting the surface density profile over the region corresponding to 0.3–0.9 of the instantaneous infall radius.

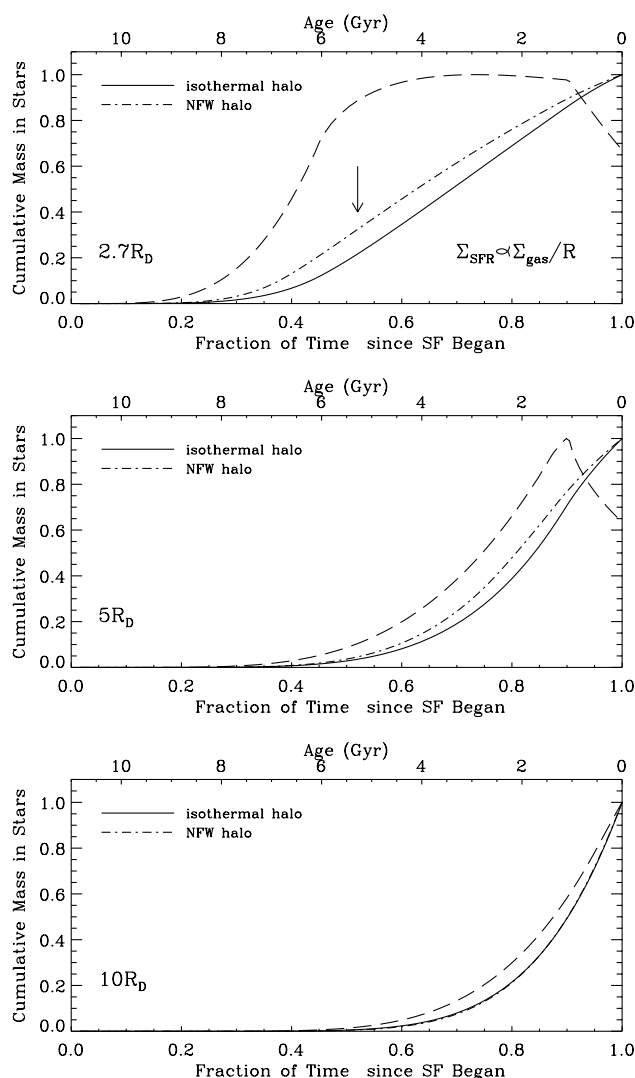
isothermal halo model in which the total infall rate is allowed to decrease exponentially with time. In this case, the amount of recent infall (i.e.  $t \sim t_{\text{now}}$ ) is reduced relative to that in the constant infall case by a factor  $\approx 11 \text{ Gyr}/\tau_{\text{inf}}$ , where 11 Gyr is the present age of

the disc and  $\tau_{\text{inf}}$  the e-folding time of the infall rate. In the limit of  $\tau_{\text{inf}} \gg 11 \text{ Gyr}$ , this infall prescription mimics the constant infall case whereas for  $\tau_{\text{inf}} \ll 11 \text{ Gyr}$  it mimics the pre-assembled disc case, with essentially all the infall occurring early on; Fig. 7 shows an intermediate case where  $\tau_{\text{inf}} = 5 \text{ Gyr}$ . We also examine an infall model in which the baryons in the halo are described by a Navarro–Frenk–White (NFW) profile in which  $\rho \propto (r/r_s)^{-1} \times (1 + r/r_s)^{-2}$  with the parameter  $r_s$  defining the scale radius. Navarro, Frenk & White (1997) argue on the basis of high resolution  $N$ -body simulations that this form better characterizes the equilibrium density profiles of dark-matter haloes than an isothermal sphere. In terms of infall rates, it leads to a somewhat higher infall rate than the isothermal sphere at early times and somewhat lower rate at late times. Fig. 7 indicates that the growth of the scalelength in these alternate infall models is very similar to that in the constant infall case and thus that our results on the scalelength evolution are robust. Contrasting Figs 6 and 7 with Fig. 3, one concludes that if clear evidence were to emerge for an increase in stellar scalelength of galaxies over cosmological time, it would, within the framework of our model, be an *unambiguous indicator of the late infall of high angular momentum material* and could not merely be attributed to the effects of viscous evolution.

A consistent picture emerges from analysis of the star formation history at different locations in the disc. Fig. 8 shows the SFR (dashed line) normalized to the peak rate as a function of time for our default star formation law. Very few stars form initially at the solar neighbourhood as a direct result of the low gas density (only gas viscously transported from smaller radii is present). As the peak infall rate at this radius is reached (indicated by the arrow in the top panel of Fig. 8), the SFR rapidly increases and then levels off to an almost constant value until the present epoch. The fraction of the final stellar mass in place as a function of time for singular isothermal sphere and NFW halo baryon distributions is also shown (solid and dashed–dotted lines, respectively). As expected, the disc at 2.7 scalelengths is predominantly young in the extreme infall model, with only a small dependence on the specific form of the infall prescription. Specifically, 50 per cent of the final stellar mass in place after 0.7 of the star-forming duration, or  $\sim 8 \text{ Gyr}$  if a disc age of 11 Gyr is assumed (note that in this case very few 11 Gyr stars are expected to exist outside the innermost regions of the disc, where the first infall is received). At 5 and 10 scalelengths, the stellar population is conspicuously young in the extreme infall model. The disc at 5 scalelengths receives material from direct infall of halo gas only at the very end of the infall phase, so that prior to this, gas only reaches this location as a result of outward viscous transport from an initially much more compact gas distribution. This is especially true of the disc at 10 scalelengths, which never receives direct infall in our model. Consequently, the dominant star-forming phase in these regions only gets under way at late times. We draw attention to the fact that this combined infall/viscous model may produce an unusual chemical signature in the very outer regions, comprising an older metal rich population (formed from pre-enriched gas transported outwards from small radii at early times) and a young metal poor population containing stars formed from recent unenriched infall. We plan to investigate detailed predictions for the chemical signatures of viscous evolution and infall in a future paper.

## 4 DISCUSSION

We have explored models for the evolution of disc galaxies subject to star formation, viscous redistribution of angular momentum, and

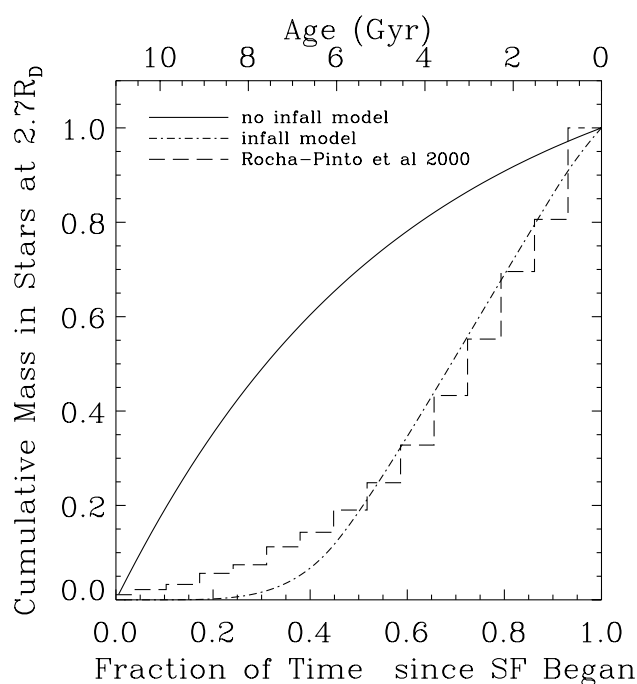


**Figure 8.** The star formation history (dashed line) of the disc at radii corresponding to 2.7 (top), 5 (middle) and 10 (bottom) exponential scalelengths in the extreme infall case. The SFR is normalized to the peak rate at each radius. These models are for a star formation law  $\Sigma_{\text{SFR}} = \Sigma_{\text{gas}}/R$ . The top axis shows the age of the disc, assuming the present-day age of 11 Gyr (Binney et al. 2000). Also shown is the fraction of the final stellar mass in place for the particular cases of baryonic infall from haloes with isothermal and NFW forms (solid and dashed-dotted lines, respectively) as a function of time. The time corresponding to the peak infall rate at the solar neighbourhood is marked by the arrow in the top panel.

cosmologically-motivated gaseous infall. We have focused attention on the extreme cases of these models and the rather different predictions which arise for the temporal evolution of the exponential scalelength, and the radial growth rate of galactic stellar discs. In this section, we discuss how these model predictions compare with relevant observational data for both local and distant disc galaxies.

#### 4.1 Constraints from the local Universe

The resolved stellar populations in the Milky Way and other nearby spirals provide the ‘fossil record’ of star formation and galaxy assembly over cosmic lifetimes. The wealth of high quality data



**Figure 9.** The cumulative mass in stars as a function of time in the solar neighbourhood as derived from the chromospheric age distribution of Rocha-Pinto et al. (2000) (dashed line). Also shown are the predictions for our extreme infall model (dashed-dotted line) and the pre-assembled disc model (solid line).

available for stars in our local neighbourhood of the Milky Way disc makes it a logical starting point for model comparisons.

An important prediction of our models is the star formation history – which is reflected in the stellar age distribution – at various radii. The pre-assembled and extreme infall models make very different predictions for the star formation history of the disc at 2.7 exponential scalelengths. In the former case, the SFR declines smoothly by a factor of  $\sim 5$  over the lifetime of the disc, with the consequence that a significant fraction of disc stars form early on. In the latter case, however, very few stars form initially with the rate then increasing steeply to a roughly constant value. As a consequence, most stars form late in this model. Quantitatively, for an assumed disc age of 11 Gyr, more than 50 per cent of the final stellar mass is in place at solar neighbourhood after  $\sim 3.5$  Gyr in the pre-assembled model whereas less than 5 per cent is in place by same time in the extreme infall model.

It is well established that stars with a considerable range in ages exist in our local neighbourhood, a non-negligible fraction of which appear to be ‘old’, i.e. ages  $\geq 8$  Gyr (e.g. Edvardsson et al. 1993; Binney et al. 2000; Rocha-Pinto et al. 2000). Rocha-Pinto et al. (2000) have recently redetermined the star formation history of the local Galactic disc from a sample of 552 late-type dwarfs exhibiting chromospheric activity. Stellar ages are derived using a new metallicity-dependent chromospheric activity–age relation (see their table 3). Assuming this sample provides an unbiased representation of the total thin disc population, we use their age distribution to infer the cumulative mass in stars present in the solar neighbourhood as a function of time, as shown by the dashed line in Fig. 9. This curve shows a surprisingly good agreement with the predictions of our extreme infall viscous model. While the Rocha-Pinto data suggest a slightly higher SFR at early times (hence a small excess of very old stars), the overall similarity of the two star formation histories at later times is quite striking. We stress that our

star formation history is largely determined by the infall history at this radius, and not by viscous flows. Although the possibility remains that some observational biases exist within the Rocha-Pinto sample – for example, the onset and duration of stellar chromospheric activity is poorly understood (e.g. Baliunas & Vaughan 1985) – it seems unlikely that these biases could alter the derived star formation history enough to make it more closely resemble the predictions of our pre-assembled model.

Knowledge of the age distributions of stars at large radii in discs (for example, at 5 scalelengths) would have even greater leverage for model comparisons, since it is precisely these parts where the predictions of the extreme infall and pre-assembled models deviate the most. Unfortunately, essentially no information is currently available on the range of stellar ages present in these parts. Due to our location within the Milky Way disc, this issue may be more easily addressed in the future by deep studies of the resolved stellar populations in nearby external galaxies than in our own.

The effects of angular momentum transport alone mean that viscous models naturally predict inside-out growth for galactic discs, whether or not infall is also present. Associating the ‘mean age’ of the stellar population with the age by which 50 per cent of the final stellar mass is in place, our models suggest an age gradient of  $\sim -0.7$  Gyr over the radial range spanning 2–5 scalelengths in discs (see Figs 4 and 8). Inside-out formation has long been suspected in disc galaxies on the basis of radial colour and metallicity gradients (e.g. Tinsley & Larson 1978). For example, de Jong (1996b) finds that radial colour gradients (in the sense of bluer colours at larger radii) extending over several scalelengths are common in spirals and are largely consistent with trends in stellar age alone. Using stellar population synthesis models, Bell & de Jong (2000) have explicitly quantified the radial age variation implied by these colour gradients and find the typical large spiral to exhibit an age gradient of  $\sim -0.8$  Gyr per  $K$ -band scalelength, which is also in good agreement with the predictions of our models.

Finally, we note that there is still significant debate as to the importance of continued gaseous infall for disc galaxy evolution, with few existing direct observational constraints. Galactic high velocity clouds may be a signature of gas accretion onto the Milky Way, but the lack of information concerning the distances of these clouds hinders a clear understanding of their origin. Blitz et al. (1999) have carried out a simple simulation of the formation and evolution of the Local Group in which  $10^6$  particles (each representing a  $10^6 M_\odot$  gas cloud) are subject to the gravitational forces of M31, the Milky Way and the tidal field of the other Local Group members. Any cloud falling towards the Milky Way or M31 which passes within 100 comoving kpc of their centres is assumed to be accreted by that galaxy. They find present-day neutral gas accretion rates of  $\sim 1 M_\odot \text{ yr}^{-1}$  for both galaxies, with the rates being somewhat higher in the distant past. The overall evolution is consistent with an exponential decline with an e-folding time of about  $5 \times 10^9$  yr. By comparison, our extreme infall model, when normalized to the Milky Way, requires a constant infall rate of  $4.5 M_\odot \text{ yr}^{-1}$ . Our default infall models therefore require gas accretion rates that are slightly higher than but of the same order of magnitude as those found in Blitz’s simulation.

## 4.2 Constraints from distant galaxies

An exciting recent development has been the availability of quantitative structural parameters for moderate redshift disc galaxies which can be used to directly map the size evolution of galaxies with cosmic epoch (e.g. Lilly et al. 1998; Roche et al. 1998; Simard et al.

1999; Giallongo et al. 2000). Based on a sample of discs drawn from the Canada–France and Low-Dispersion Spectrograph redshift surveys, Lilly et al. (1998) find the size function of disc scalelengths in disc-dominated galaxies (i.e. bulge-to-total ratios  $\leq 0.5$ ) to be roughly constant to  $z \sim 1$ , at least for large discs with  $R_D > 3.2 h_{50}^{-1}$  kpc. In a similar study, Simard et al. (1999) analyse the size–magnitude relation defined by disc-dominated field galaxies in the Deep Extragalactic Evolutionary Probe survey. Carefully considering possible selection effects, these authors also conclude that there is no significant evidence for any evolution in the size–magnitude distribution of disc galaxies over the redshift range  $0.1 < z < 1.1$  and that a significant number of discs lie close to the canonical Freeman relation at all redshifts probed.

Taken together, these studies suggest very little change in stellar exponential scalelength of disc galaxies with epoch out to  $z \sim 1$  (however see Mao, Mo & White (1998) for an alternative interpretation of the Lilly data set). For an Einstein–de Sitter Universe, the interval between a redshift of unity and the present epoch can be associated with the period from  $\sim 0.3$  to 1.0 times the star-formation interval in our models, assuming that star formation in the disc begins early on. Inspection of Figs 6 and 7 indicates that the disc scalelengths are expected to increase by about a factor of 3 over this redshift range if gaseous infall is important. Thus, in contrast to the Milky Way constraints discussed in the previous section, it would appear that observations of moderate redshift galaxies lend support for a scenario in which prolonged, angular momentum-conserving infall is *not* important for building galactic discs. We note that the global SFR of the discs which form in our pre-assembled and extreme infall models decrease and remain roughly constant with time, respectively – neither of which are inconsistent with current observations of large disc galaxies.

It should be kept in mind, though, that quantitative characterization of distant galaxy profiles is still plagued by a number of difficulties. For the highest redshift galaxies in the aforementioned samples, exponential profiles are generally fit over 1–2 disc scalelengths and hence are particularly susceptible to errors in the bulge and disc separation. Furthermore, studies to date have measured structural parameters in a single passband over the redshift range of interest. If strong colour gradients are present in these systems, this could conspire with the  $k$ -correction to mimic a constant scalelength in a fixed passband over a range in redshift even though the rest-frame scalelengths are evolving. Colour gradients significantly in excess of those measured in local galaxies would be required for this to be an important effect however.

## 5 CONCLUSIONS

We have explored models for the evolution of disc galaxies that invoke simultaneous star formation and viscous redistribution of angular momentum, as well as a cosmologically-motivated prescription for gaseous infall. The existence of galaxies which exhibit exponential disc profiles over significantly more than  $\sim 5$  scalelengths (e.g. Barton & Thompson 1997; Weiner et al. 2000) lend strong support for viscous models as viscosity may be the only viable mechanism for transporting material to these extreme radii, while maintaining a smooth exponential profile in the stellar disc. Following Lin & Pringle (1987a), we fix the relationship between the star formation and viscosity prescriptions such that the characteristic time-scale for material to move a fractional radius of order unity is comparable with the time-scale on which it is converted into stars (i.e.  $t_* \sim t_\nu$ ). While the inclusion of viscous flows is essential for ensuring that the stellar disc is always

exponential, we find that such flows play only a minor role in determining the evolution of the disc scalelength, and the star formation history over the inner  $\sim 5$  scalelengths (corresponding to the maximum infall radius in our model). Indeed, the dominant process governing the size evolution and star formation history of the luminous stellar disc is that of gaseous infall.

We summarize our main results as follows:

(i) In models in which the main infall phase precedes the onset of star formation (i.e. a pre-assembled gas disc), we find the exponential scalelength to be rather *invariant* with time over the entire history of the disc. This result holds true for a variety of popular star formation prescriptions. The value of the stellar disc scalelength is primarily determined by mean specific angular momentum of the initial gas disc.

(ii) In the pre-assembled model, the SFR smoothly declines with time within the inner 5 scalelengths (approximately the maximum infall radius,  $r_{\text{infall}}$ ). As a result, there is a considerable ‘old’ (i.e.  $> 8$  Gyr) population present at these radii, with half of the final stellar mass in place more than 6 Gyr ago. The onset of star formation is delayed at radii greater than  $r_{\text{infall}}$  as the outer disc is formed entirely by material viscously transported from smaller radii; the stellar populations in these parts are thus increasingly young with radius.

(iii) If infall continues during the main star-forming phase of the galaxy, the stellar disc remains approximately exponential at all times, but with the scalelength *increasing towards the present epoch*, in simple proportion to the average specific angular momentum of the infallen material. For an isothermal halo density profile and constant spin parameter, the scalelength is directly proportional to the infallen mass which is proportional to time. Regardless of whether viscosity is at work or not, late infall of high angular momentum material is the only way to significantly increase the stellar scalelength of discs in our models.

(iv) In contrast to the case of a pre-assembled disc, the disc stellar populations in the infall models are predominantly young, with more than half of the stars at the location of solar neighbourhood predicted to have ages  $\lesssim 5$  Gyr. The disc at 5 scalelengths receives direct infall of halo gas only at the very end of the accretion phase, so that prior to this, gas only reaches this location as a result of outward viscous transport. As in the pre-assembled case, all material at radii beyond the maximum infall radius has been viscously transported there from smaller radii.

(v) We compare our model predictions with recent observations of disc galaxy sizes and stellar populations. A particularly good agreement is found between the solar neighbourhood star formation history predicted by our extreme infall model and the recent observational determination of this quantity by Rocha-Pinto et al. (2000). On the other hand, the relative invariance of disc scalelength with cosmic epoch, as derived from studies of moderate redshift galaxies, appears to lend support to our pre-assembled model. Future observations that can further test the models presented here include detailed studies of resolved stellar populations at large galactocentric radii ( $\geq 5$  scalelengths) in the Milky Way and other nearby spirals in order to constrain the age and star formation history of the outer disc, and larger, deeper surveys of the structural parameters of moderate redshift disc systems in various passbands.

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